Questions – Week 4: The Gravitational Path Integral and Thermodynamics of de Sitter Space

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Chapter 6: The Gravitational Path Integral

1. (Jerome) Why does the boundary condition for the thermal partition function $Z(\beta)$ includes $g_{tt} \to 1$ as $r \to \infty$, i.e. why is $t_E \sim t_E + \beta$ not sufficient in Eq. (6.2)?

   ANS: I would say that the two things are unrelated. The periodicity only comes from considering Euclidean time and it would not be there if we were considering 'normal' time. The second condition is just what one would expect of the geometry at infinity: being flat.

2. (Guilherme) Can we discuss a little bit about the meaning of taking time to be complex in the context of dynamical spacetime? When spacetime is fixed, I see a Wick rotation as a trick that help us to compute correlators, for example. Now, when gravity is dynamical, that means it follows Eintein's equations. Doing a Wick rotation means changing the metric signature, which implies changing it determinant from 1 to -1 and vice-versa. This passes through a singularity and I don't understand how this could possibly be well defined.

3. (Evan) What are the symmetries of the action (6.6)?

   Equations 19.50-19.54 of "Springer Handbook of Spacetime" (see http://link.springer.com/book/10.1007download it for free)" show that invariance under boundary diffs leads to a set of conserved charges.

   How many conserved charges do I get? I would guess that I would get one conserved charge for every boundary isometry. Is this correct? Furthermore, it seems that your choice of embedding for ∂M should change the number of conserved charges, and there should exist some embeddings which have NO conserved charges.

   Can we talk about bulk isometries vs. boundary isometries?

   ANS: It depends on the boundary. In the end, the symmetries will be all the isometries that are not broken by the boundary. However, if you are doing local physics (scatterings), then you probably do not care about it.

   And yes, if you change the manifold in which you consider your boundary, you have more or less isometries.

   Bulk/local vs boundary/global symmetries is a very delicated issue. It is common that the imposition of different boundary conditions breaks some of the local symmetries you have at the level of equations of motion. This is a relevant issue especially when one is worried to define properly the phase space before quantizing the system.

4. (Yan) I think I should know that but I don't see why Euclidean Schwarzschild has no horizon. The components of the metric still vanish at the "horizon".

   ANS: Euclidean space has only positive norm, therefore $r \geq 2M$.

5. (Yan) What is the physical meaning of the counterterm? In QFT it means something to add them because of renormalization but here it seems to be an ad hoc addition. Specifically we could add any constant we want to this counterterm and change the answer.

   ANS: It just means that you started with the wrong action. As in QFT, you discover that the action you started with is made of non-physical parameters and then you sum the counterterms. In principle, if you knew about that, you could have started with the "correct" action.

   The second part of the question is very interesting. Indeed, you can add any constant, but it must be coupled to the metric. That means, you are really introducing a cosmological constant and, therefore, changing the asymptotic spacetime. This has several implications (my seminar at Ian's course was about BH in De Sitter backgrounds - weird things happen! don't remember quite well now :P). In summary, not allowed if you want asymptotically flat spacetime.

6. (Yan) I don't think I see the diference between the boundary term and the counterterm.

7. (Max) When deriving the counterterm in 6.14, it says "K_0 is the extrinsic curvature of the same boundary manifold M, embedded in flat spacetime." What does it mean for two boundary manifolds to be "the same" when they're embedded in different spacetimes?

   "This is very similar to what we do in quantum field theory, but this calculation is entirely classical." Could we spell out this "similarity" more explicitly?

8. (Yan) Just to be sure: the Bekenstein-Hawking entropy and all of that are only at first order right? Because otherwise it's weird that an approximation of the partition function gives the total answer.

   ANS: Yes, they come from the saddle point approximation (classical solution -¿ fixed metric).

Chapter 7: Thermodynamics of de Sitter space

1. (Jerome) I don't understand Fig. (7.4). In fact, I don't understand the direction of the Euclidean time. I thought that it went from the pole to the equator where the vacuum is prepared, or am I thinking the wrong way?

   ANS: You are right, that why it says ON the equator. For instance, when we were studying flat spacetime, the integral would go from -∞ to 0, preparing a vacuum state at t=0.

2. (Jerome) How is it that "most states will eventually end up close to the Euclidean vacuum, since the de Sitter expansion dilutes any excitations?" In inflation, de Sitter doesn't "dilute" the vacuum, but rather it "stretches" it to become the large scale structures of our universe.

   ANS: I guess you are both saying the same, basically.

3. (Jerome) Can someone clarify for me why it is that Eq. (7.7) is valid for an on-shell action and how one finds that this on-shell action is Eq. (7.8)?

   ANS: ... On-shell action just means using e.o.m at the action level. Then, one can easily see how equation (7.8) came to be.

4. (Jerome) Is Eq. (7.10) correct? In cosmology, we usually expect

$$h_k'' + \left(k^2 - \frac{2}{\eta}\right) h_k = 0 \ .$$

   Or is this different?

   ANS: Are you sure? Because the dimensions are not right in your equation.

5. (Jerome) What happened to the term ik^2/η_0 in going from Eq. (7.13) to Eq. (7.14)?
   (Evan) I'm confused with the manipulation in 7.14. I thought to derive this result you needed to use the Bunch-Davies initial condition: $\phi_k^0 = e^{ik\eta}/\sqrt{2k}$, which is applied at $|k\eta| >> 1$ (i.e. early times, $\eta \geq -\infty$, or small length scales $k \geq \infty$). He doesn't appear to use this anywhere. This would imply that any initial state will evolve towards a scale-invariant spectrum. Is this true? Am I missing something?

   ANS: I believe those two questions are the same. Exactly because one needs $\eta \to \infty$, the term ik^2/η_0 is gone.

6. (Leila) page 75: "The static patch is the region of de Sitter in causal contact with an observer sitting at the north pole. This is the analogue of the Rindler patch." Why ?

7. (Yan) Since we are now in a de Sitter era, can we observe the temperature associated with it?

   ANS: No, the temperature is amazingly small. It has to be proportional to the Hubble constant. If one recovers the fundamental constant to match the dimensions and plug in the numbers, it is of order $10^{-30}K$.